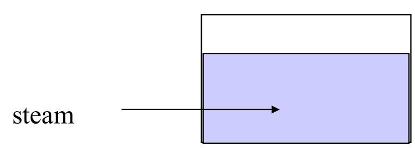
Energy Balance : Steady State Flow

- no changes in mass of the system *and* no changes with time in the properties of the fluid within the c.v. or its entrances and exits
- only shaft work is possible
- the rates at which heat and work cross the control surface is constant

$$\dot{Q} + \dot{W} = \Delta\left[\left(H + \frac{u^2}{2} + zg\right)\dot{m}\right]_{fs}$$

Example

Water in a metal vessel is heated from 25 °C (U = 104.88 kJ/kg) to 80 ° C (U = 334.88 kJ/kg) by adding steam slowly enough that all the steam condenses in the water. The vessel initially contains 100 kg of water and the entering steam (300 ° C and 3 atm) has an enthalpy of 3069.3 kJ/kg. How much steam needs to be added? Assume the tank is well insulated and has a diameter that is very much greater than the steam inlet pipe.



Define the surface of the control volume as the surface of the body of water in the tank. Stream 1 is entering and there is not Stream 2

Given: Initial m_i = 100 kg T_i = 25ºC U_i = 104.88 kJ/kg

Final m_f = required to find = $m_i + m_1$ T_f = 80ºC U_f = 334.88 kJ/kg

Steam added slowly has H_s = 3069.3 kJ/kg at 3 atm and 300ºC

Well insulated and large diameter tank

First do a mass balance: $\Delta m_{cv} = m_1 = m_s$ (mass of steam added)

Energy balance:

$$\frac{d(mU)_{cv}}{dt} = \dot{Q} + \dot{W} - \Delta\left[\left(H + \frac{u^2}{2} + zg\right)\dot{m}\right]_{fs}$$

Now from the statement of the problem:

Well insulated → $\dot{Q} = 0$
No shaft work → $\dot{W}_S = 0$
Nothing is leaving the tank → $m_2 = 0$
For a large diameter tanks the change of height in the tank is small Δz is negligible
Adding steam very slowly therefore Δu^2 is negligible

This leaves:

$$\frac{d(mU)_{cv}}{dt} = -\Delta\left[(H)\dot{m}\right]_{fs} = \left(H_2\dot{m}_2 - H_1\dot{m}_1\right) = \left(H_S\dot{m}_s\right)$$

Integrating from m_i to m_f over time

$$\int_{m_iU_i}^{m_fU_f} d(mU)_{cv} = m_fU_f - m_iU_i = H_S\int_{t_i}^{t_f}\dot{m}_s dt = H_S m_s$$

From the mass balance:

$m_f = m_i + m_s = 100 + m_s$

From energy balance

$$(100+m_s)U_f - 100\ U_i = H_S m_s$$

Solve for m_s

$$100U_f + m_sU_f - 100\ U_i - H_S m_s = 0$$
$$m_s\left(U_f - H_S\right) + 100\left(U_f - U_i\right) = 0$$

$$m_s = 100\frac{\left(U_i - U_f\right)}{\left(U_f - H_S\right)} = 100\frac{(104.88 - 334.88)}{(334.88 - 3069.3)} = 8.4\ \text{kg}$$