

**CHEE210 Winter 2011 Quiz #2**

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**This test is closed book.** All data required to answer the problems are included with the quiz paper. Answer all questions on the pages provided. Use the back of pages if necessary being sure to mark the problem number and section on the additional work. **Write your name and student number on each page.** There are three (3) problems and four (4) single-side pages. Complete all problems.

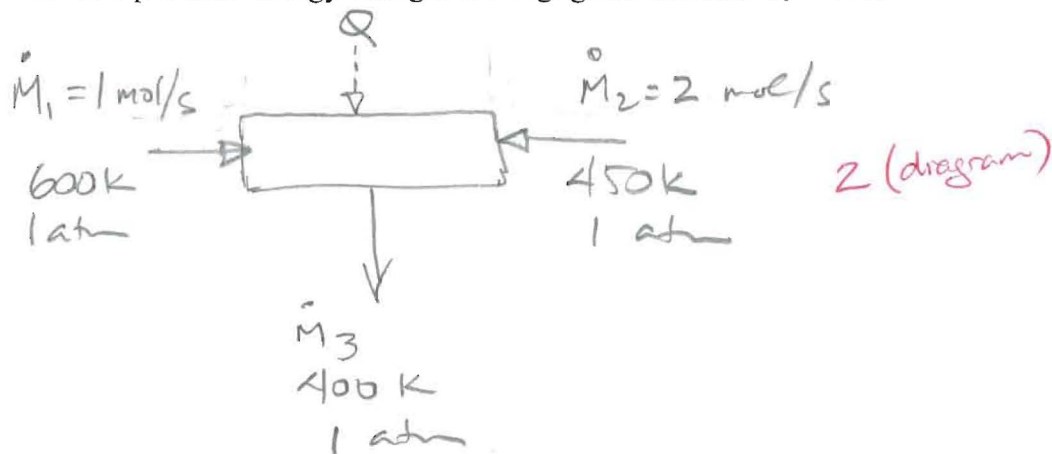
(Time: 50 minutes)

**Useful Numbers (watch your units!!)**

1 atm = 101.4 kPa

$R = 8.314 \text{ J mol}^{-1} \text{ K}^{-1} = 8.314 \text{ L kPa mol}^{-1} \text{ K}^{-1} = 82.06 \text{ cm}^3 \text{ atm mol}^{-1} \text{ K}^{-1} = 0.08206 \text{ L atm mol}^{-1} \text{ K}^{-1}$

1. In a steady-state process, 1 mol/s of air at 600 K and 1 atm is continuously mixed with 2 mol/s of air at 450 K and 1 atm. The product stream is at 400 K and 1 atm. Determine the rate of heat transfer and the rate of entropy generation for this process. The surroundings are at 300 K. Assume that kinetic and potential energy changes are negligible. Assume  $C_p = 7/2R$

Marks  
(15)

Mass Balance (Steady State)

$$\dot{n}_3 + \dot{n}_2 + \dot{n}_1 = 0$$

$$\dot{n}_3 = -(\dot{n}_2 + \dot{n}_1) \quad | \text{ (mass bal)}$$

Energy Balance (S.S.)

$$\dot{Q} + \cancel{\dot{W}} = \Delta \left[ \left( H + \frac{u^2}{2} + zg \right) \dot{n} \right] \quad | \text{ (energy bal)}$$

$$\dot{Q} = [\dot{n}_3 H_3 - \dot{n}_2 H_2 - \dot{n}_1 H_1]$$

subst. mass balance.

$$\dot{Q} = [(\dot{n}_2 + \dot{n}_1) H_3 - \dot{n}_2 H_2 - \dot{n}_1 H_1]$$

$$= \dot{n}_1 (H_3 - H_1) + \dot{n}_2 (H_3 - H_2)$$

$$= \dot{n}_1 c_p (T_3 - T_1) + \dot{n}_2 c_p (T_3 - T_2)$$

$$= 1 \left( \frac{7}{2} R \right) (400 - 600) + 2 \left( \frac{7}{2} R \right) (400 - 450)$$

$$\dot{Q} = -8.73 \frac{\text{kJ}}{\text{s}} = -8.73 \text{ kW}$$

Entropy Balance

$$\Delta(\dot{n}S)_{fs} = \sum \frac{\dot{Q}_j}{T_{\sigma, j}} + S_{gen}$$

$$-\dot{n}_1 S_1 - \dot{n}_2 S_2 + \dot{n}_3 S_3 = \frac{\dot{Q}}{T_{\sigma}} + S_{gen}$$

$$-\dot{n}_1 S_1 - \dot{n}_2 S_2 + (\dot{n}_1 + \dot{n}_2) S_3 = \frac{\dot{Q}}{T_{\sigma}} + S_{gen}$$

$$\dot{n}_1 (S_3 - S_1) + \dot{n}_2 (S_3 - S_2) = \frac{\dot{Q}}{T_{\sigma}} + S_{gen}$$

1 (entropy bal with mass bal)

1 (converting to  $\Delta S$  form)

$$\dot{n}_1 C_p \int_{T_1}^{T_3} \frac{dT}{T} + \dot{n}_2 C_p \int_{T_2}^{T_3} \frac{dT}{T} = \frac{\dot{Q}}{T_0} + S_{gen}$$

*(correct integration)*

$$\dot{n}_1 \left(\frac{7}{2}R\right) \ln \frac{T_3}{T_1} + \dot{n}_2 \left(\frac{7}{2}R\right) \ln \frac{T_3}{T_2} = \frac{\dot{Q}}{T_0} + S_{gen}$$

$$\frac{7}{2} (8.314) \left[ (1) \ln \left(\frac{400}{600}\right) + 2 \ln \left(\frac{400}{450}\right) \right] = \frac{-8730}{300} + S_{gen}$$

*(subbing in all values)*

$$-18.6 = -29.1 + S_{gen}$$

$$S_{gen} = 10.5 \frac{J}{K \cdot s}$$

*2 (final answer with units correct)*

2. For Propane at 70°C and 1500 kPa estimate  $H^R$  and  $S^R$  using the Generalized Pitzer Correlation (see equation sheet).

$P_C = 42.48 \text{ bar}$ ,  $T_C = 369.8 \text{ K}$  and  $\omega = 0.152$

$$P_R = \frac{1500}{4248} = 0.353 \quad T_R = \frac{343.15}{369.8} = 0.928 \quad (9) \text{ (reduced variables)}$$

$$\frac{H^R}{RT_C} = P_R \left[ B^0 - \frac{0.675}{T_R^{1.6}} + \omega \left( B^1 - \frac{0.722}{T_R^{4.2}} \right) \right] \quad (1) \text{ (correct equation with } T_C)$$

$$B^0 = 0.083 - \frac{0.422}{T_R^{1.6}} \quad B^1 = 0.139 - \frac{0.172}{T_R^{4.2}}$$

$$= -0.3925 \quad = -0.0964$$

$$\frac{H^R}{T_C} = 0.353 \left[ -0.3925 - \frac{0.675}{(0.928)^{1.6}} + 0.152 \left( -0.0964 - \frac{0.722}{(0.928)^{4.2}} \right) \right]$$

$$= -0.4653$$

$\frac{1}{2}$  (intermediate ans)

$$H^R = (369.8 \text{ K})(-0.4653) \left( 8.314 \frac{\text{J}}{\text{mol K}} \right)$$

$$= -1431 \frac{\text{J}}{\text{mol}} \quad (2) \text{ (final ans w units)}$$

$$\frac{S^R}{R} = -P_R \left[ \frac{0.675}{T_R^{2.6}} + \omega \left( \frac{0.722}{T_R^{5.2}} \right) \right] = 0.347 \quad (1) \text{ (correct equation)}$$

$\frac{1}{2}$  (intermediate answer)

$$S^R = -0.347 \times 8.314 \frac{\text{J}}{\text{mol K}} = -2.881 \frac{\text{J}}{\text{mol K}}$$

$\frac{1}{2}$  (final answer by R multiply)

3. Using the Gibbs Equations, the Maxwell's Equations and the basic definition of  $C_p$  (given on the attached equation sheet) derive the formula (6)

$$dS = C_p \frac{dT}{T} - \left( \frac{\partial V}{\partial T} \right)_P dP$$

Recall that we can write the following expression for  $dS$

$$dS = \left( \frac{\partial S}{\partial T} \right)_P dT + \left( \frac{\partial S}{\partial P} \right)_T dP \quad \dots \dots \textcircled{A}$$

From Gibbs Equations

$$dH = TdS + VdP \quad | \quad (\text{choosing Gibbs Equ'n})$$

For const  $P$ ,  $dP = 0$

$$dH = TdS \quad | \quad (\text{clearing } P \text{ with } dP = 0)$$

Divide through by  $dT$  and use definition of  $C_p$

$$\left( \frac{\partial H}{\partial T} \right)_P = T \left( \frac{\partial S}{\partial T} \right)_P = C_p \quad | \quad (\text{dividing by } dT \text{ using } C_p \text{ definition})$$

Re-arranging, (solving for differential)

$$\left( \frac{\partial S}{\partial T} \right)_P = \frac{C_p}{T} \quad | \quad \textcircled{B}$$

From Maxwell's Equations

$$\left( \frac{\partial S}{\partial P} \right)_T = - \left( \frac{\partial V}{\partial T} \right)_P \quad \textcircled{C}$$

(using Maxwell's relation)

Substitute  $\textcircled{B}$  and  $\textcircled{C}$  into  $\textcircled{A}$  to obtain! (correct substitution)

$$dS = C_p \frac{dT}{T} - \left( \frac{\partial V}{\partial T} \right)_P dP$$

Q1 \_\_\_\_\_

Q2 \_\_\_\_\_

Q3 \_\_\_\_\_