

CHEE210 Winter 2008 Quiz #1

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Date: Thursday, 31 January 2008.

Closed book. All thermodynamic equations you will need are included in the statement of the problem. Answer all questions on the pages provided. Use the back of pages if necessary being sure to mark the problem number and section on the additional work. Write your name and student number on each page. (Time: 60 minutes)

(Values for $R = 8.314 \text{ J mol}^{-1} \text{ K}^{-1} = 82.06 \text{ cm}^3 \text{ atm mol}^{-1} \text{ K}^{-1} = 0.08206 \text{ L atm mol}^{-1} \text{ K}^{-1}$)

Marks

1. Each morning Dr. Peppley wakes up, gets dressed in a dashing athletic outfit and works out for 20 minutes on his Infiniti elliptical trainer equipped with a watt meter and cumulative calorie meter. On a typical day, Dr. Peppley generates power at a rate of 180 watts during the workout and the calorie accumulator registers that 320 calories of energy have been produced at the end of 20 minutes.

- a) How many joules of energy did Dr. Peppley actually generate in the 20 minute period based on the watt meter? (5)

$$Power = 180 \text{ W} \quad 180 \text{ J s}^{-1} \times 20 \text{ min} \times 60 \text{ s min}^{-1} = 216 \text{ kJ} \text{ of energy produced}$$

$$\text{or} = 216,000 \text{ J of energy produced}$$

- b) A calorie is approximately equivalent to 4.18 J. Is the calorie accumulator correct? What should it read? (5)

$$Energy \text{ in Cal} = \frac{216 \text{ kJ}}{4.18 \text{ J cal}^{-1}} = 51.7 \text{ kcal}$$

The accumulator read 320 calories whereas 51.7 kcal were generated it is giving an incorrect reading.

- c) During the months of December and January last year Dr. Peppley's electrical bill indicated that 1650 kWh were consumed over a 60 day period. How many hours would Dr. Peppley need to workout on the elliptical trainer to provide one average day's worth of electrical energy for his house? (5)

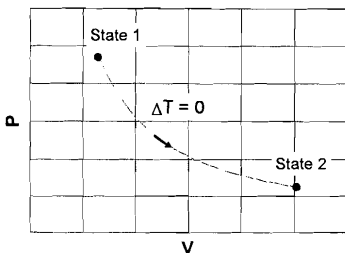
$$Total \text{ energy per day} = \frac{1650 \text{ kWh}}{60 \text{ days}} = 27.5 \frac{\text{kWh}}{\text{day}}$$

$$= 99,000 \text{ kJ.}$$

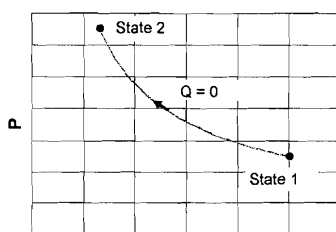
$$Seconds \text{ Required} = \frac{99,000}{0.180} = 550,000 \text{ s}$$

$$= 153 \text{ hrs.}$$

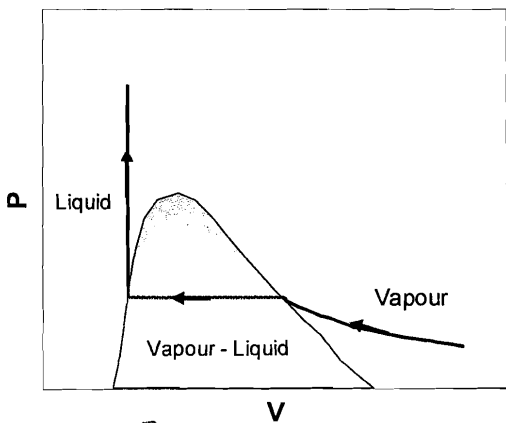
2. For each of the following P-V Diagrams give a brief description of the process that is occurring (e.g., reversible constant pressure expansion.).



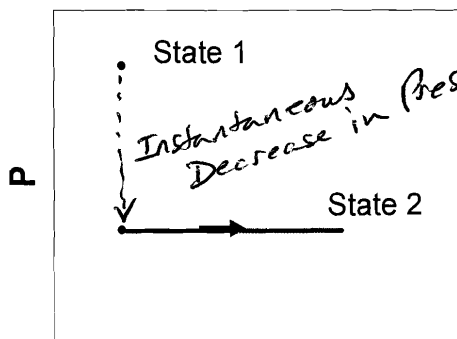
(a) Isothermal reversible expansion



(b) Adiabatic reversible compression



(c) Condensation at constant temperature by increasing pressure



(d) Irreversible isobaric expansion
or
Irreversible constant pressure expansion.

Assuming 1 bar = 1 atm is acceptable.

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3. (a) Calculate the molar volume of oxygen at 20°C and 20 atm using the Redlich-Kwong Equation. $T_C(O_2) = 154.6 \text{ K}$, $P_C(O_2) = 50.43 \text{ atm}$. (25)

$$P = \frac{RT}{V-b} - \frac{a}{V(V+b)}$$

$$a = 0.42748 \frac{R^2 T_C^2}{T_R^{0.5} P_C} \quad \text{and} \quad b = 0.08664 \frac{RT_C}{P_C}$$

$$a = 0.42748 \left(82.06 \frac{\text{cm}^3 \text{ atm}}{\text{mol K}} \right)^2 (154.6 \text{ K})^2$$

$$b = 0.08664 \frac{\left(\frac{293.15}{154.6} \right)^{0.5} 50.43 \text{ atm} \left(82.06 \frac{\text{cm}^3 \text{ atm}}{\text{mol K}} \right) (154.6 \text{ K})}{50.43} = 22.1 \text{ cm}^3/\text{mol}$$

$$P = \frac{\left(82.06 \frac{\text{cm}^3 \text{ atm}}{\text{mol K}} \right) \left(293.15 \text{ K} \right)}{(V - 22.1) \text{ cm}^3/\text{mol}} - \frac{990,760 \text{ cm}^6/\text{mol}^2 \text{ atm}}{V(V + 22.1) \text{ cm}^3/\text{mol}}$$

I.G.L. $V = \frac{RT}{P} = \frac{(82.06)(293)}{20} = 1202 \text{ cm}^3 \text{ mol}^{-1}$

iter	V (cm ³)	P (atm)
1	1202	19.7
2	1150	20.6
3	1180	20.1

Ans: $V = 1180 \text{ cm}^3/\text{mol}$

- (b) Based on the value of the volume calculated in part (a) what is the estimated value of the compressibility factor, Z. (recall $PV_{\text{act}} = ZRT$)? (5)

$$Z = \frac{P V_{\text{act}}}{RT} = \frac{1180}{1202} = 0.982$$

$$= \frac{V_{\text{act}}}{V_{\text{IGL}}}$$

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$$P_V = nRT$$

- 4 An ideal gas initially at 600 K and 10 bar undergoes a four-step mechanically reversible cycle in a closed system. In Step 12, pressure decreases isothermally to 3 bar; in Step 23, pressure decreases at constant volume to 2 bar; in Step 34, volume decreases at constant pressure; and in Step 41, the gas returns adiabatically to its initial state. Take $C_p = 7/2R$ and $C_v = 5/2R$.

Some Useful Equations (Note you may need to use two equations to get the equation you need):

$$\Delta U = Q + W$$

$$PV = RT$$

$$dW = -P_{\text{ext}}dV$$

Isothermal Work: $W = RT \ln\left(\frac{P_2}{P_1}\right) = RT \ln\left(\frac{V_1}{V_2}\right)$

Adiabatic Work: $W = \frac{RT_2 - RT_1}{\gamma - 1} = \frac{P_2V_2 - P_1V_1}{\gamma - 1}$

Adiabatic Equations: $\gamma = C_p/C_v$

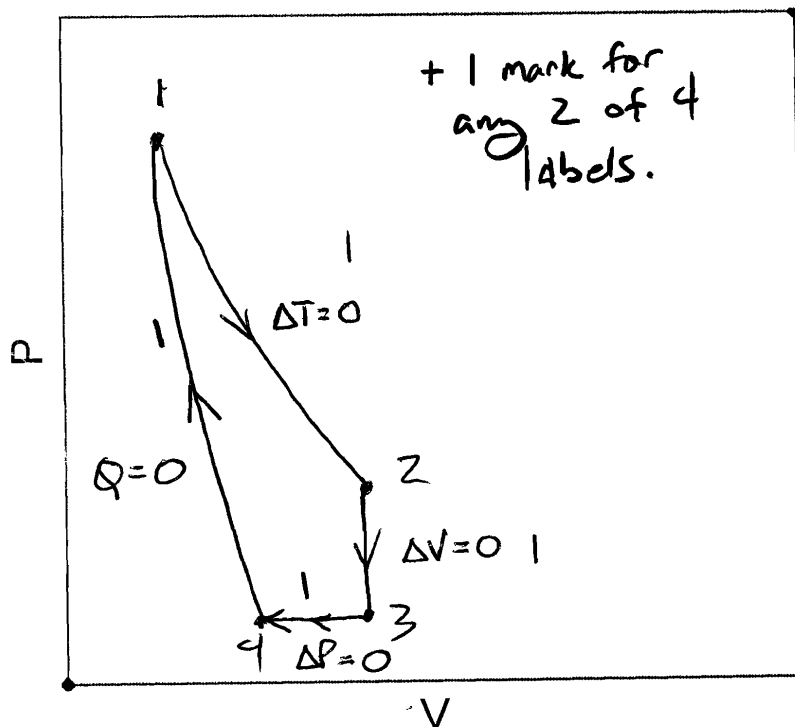
$$P_1V_1^\gamma = P_2V_2^\gamma$$

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

$$T_1P_1^{1/\gamma} = T_2P_2^{1/\gamma}$$

(a) Sketch the cycle on a PV diagram.

(5)



Work back to establish Pt. 4.

$$P_1 V_1^\gamma = P_4 V_4^\gamma \quad |$$

(or $T_1 P_1^{\frac{1-\gamma}{\gamma}} = T_4 P_4^{\frac{1-\gamma}{\gamma}}$)

$$P_4 = P_3 = 2 \text{ bar}$$

$$V_1 = \frac{RT}{P_1} = \frac{(8.314)(600)}{1000}$$

$$= 4.9884 \text{ L}$$

$$(10)(4.9884)^{1.4} = \frac{2.0}{10} V_4^{1.4}$$

$$V_4 = \left(\frac{10(4.9884)^{1.4}}{2.0} \right)^{\frac{1}{1.4}} = 15.75 \text{ L}$$

$$T_4 = \frac{P_4 V_4}{R} = \frac{200 \text{ kPa} \cdot 15.75 \text{ L}}{8.314 \frac{\text{L} \cdot \text{kPa}}{\text{mol} \cdot \text{K}}} = 379 \text{ K}$$

=
Step 34, Constant pressure compression

$$P_3 = 2 \text{ bar}, T_3 = 400 \text{ K}, V_3 = V_2 = \frac{P_1 V_1}{P_2} = \frac{(10)(4.9884)}{3}$$

$$\Delta U = C_v \Delta T = \frac{5}{2} R (379 - 400) = -437 \text{ J} \quad | \quad V_3 = 16.63 \text{ L}$$

$$\Delta H = Q_p = C_p \Delta T = \frac{7}{2} R (379 - 400) = -611 \text{ J}$$

$$\Delta U = Q + W \Rightarrow W = \Delta U - Q = -437 - (-611)$$

$$W = 174 \text{ J}$$

$$\Delta U_{34} = -437 \text{ J}$$

$$\Delta H_{34} = Q_{34} = -611 \text{ J}$$

$$W_{34} = 174 \text{ J}$$

=
Step 41/2 $Q = 0$, $\Delta U = C_v \Delta T = \frac{5}{2} R (600 - 379) = 4594 \text{ J}$

$$W = \Delta U = 4594 \text{ J}$$

$$\Delta H = C_p \Delta T = \frac{7}{2} R (600 - 379) = 6431 \text{ J}$$

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(b) Calculate Q, W, ΔU, ΔH for each step in the cycle.

Step 12 $P_1 = 10 \text{ bar}$, $T_1 = 600 \text{ K}$, $V_1 = \frac{RT_1}{P_1} = 4.9884 \text{ L}$
 $P_2 = 3 \text{ bar}$, $T_2 = 600 \text{ K}$

$$-Q = +W = RT \ln\left(\frac{P_2}{P_1}\right) = (8.314)(600) \ln\left(\frac{3}{10}\right) = -6006 \frac{\text{J}}{\text{mol}}$$

$\Delta U = 0$, $\Delta H = 0$, $Q = 6006 \text{ J/mol}$, $W = -6006 \text{ J/mol}$

Step 23 $\Delta V = 0$, $W = 0$, $V_3 = V_2$, $P_2 = 3 \text{ bar}$, $P_3 = 2 \text{ bar}$
 $T_2 = 600 \text{ K}$

$$\frac{P_2 V_2}{T_2} = \frac{P_3 V_3}{T_3} \Rightarrow T_3 = P_3 \cdot \frac{T_2}{P_2} = 2 \cdot \frac{600}{3} = 400 \text{ K}$$

$$\Delta U = C_V \Delta T = \left(\frac{5}{2}R\right)(-200) = -500 \cdot 8.314 = -4157 \text{ J/mol}$$

$$\Delta U = Q = -4157 \text{ J/mol}$$

$$\Delta H = C_P \Delta T = \left(\frac{7}{2}R\right)(-200) = -700 \cdot 8.314 = -5819.8 \frac{\text{J}}{\text{mol}}$$

Step 34 $\Delta P = 0$, $P_3 = 2 \text{ bar}$, $T_3 = 400 \text{ K}$, $V_3 = \frac{P_2 V_2}{P_3} = \frac{10 \cdot 4.9884}{2} = 24.942 \text{ L}$

(c) For the complete cycle W_{Cycle} , Q_{Cycle} , ΔU_{Cycle} , ΔH_{Cycle} .

(5)

Step 12 $Q = 6006 \text{ J/mol}$, $W = -6006 \text{ J/mol}$, $\Delta U = 0$, $\Delta H = 0$

Step 23 $Q = -4157 \text{ J/mol}$, $W = 0$, $\Delta U = -4157 \text{ J/mol}$, $\Delta H = -5820 \text{ J/mol}$

Step 34 $Q = -611 \text{ J/mol}$, $W = 174 \text{ J/mol}$, $\Delta U = -437 \text{ J/mol}$, $\Delta H = -611 \text{ J/mol}$

Step 41 $Q = 0$, $W = 4594 \text{ J/mol}$, $\Delta U = 4594 \text{ J/mol}$, $\Delta H = 6431 \text{ J/mol}$

Cycle $\Sigma Q = 6006 - 4157 - 611 = 1238 \text{ J/mol} = Q_{\text{cycle}}$
 $\Sigma W = -6006 + 0 + 174 + 4594 = -1238 \text{ J/mol} = W_{\text{cycle}}$

$\Delta U_{\text{cycle}} = 0$
 $\Delta H_{\text{cycle}} = 0$

!! for recognizing that they must be equal and opposite