Name: SOLUTION

Student No.:

CHEE210 Winter 2010

Instructor: Dr. Brant A. Peppley, P.Eng.
Date: Wednesday, 17 March 2010.

38 minutes total.

Answer all **four** questions on the pages provided. Use the back of pages if necessary being sure to mark the problem number and section on the additional work. Write your name and student number on each page. (Time: **60 minutes**. Total number of marks: 40. Plan time accordingly)

1.______/10 2.______/10 3.______/10 4.______/10 = ______/40

Temperature: **T (K) = T (°C) + 273.15**
Universal Gas Constant **R** = 8.314 J mol^{-1} K^{-1}=8.314 L kPa mol^{-1} K^{-1}
R= 82.06 cm^3 atm mol^{-1} K^{-1}=0.08206 L atm mol^{-1} K^{-1}
1 watt = 1 joule per sec 1 bar = 100 kPa
1 kWh = 1000 W × 3600 s = 3.6 MJ

Marks

1. Calculate the compressibility factor, Z, and the specific volume, V, for ethylene at 25°C and 12 bar using the truncated virial equation.

$$Z = \frac{PV}{RT} = 1 + \frac{BP}{RT}$$ (10)

Use the generalized Pitzer correlations (refer to equation sheets attached).

For ethylene from Table B of text:
$\omega = 0.087$, $T_C = 282.3$ K, $P_C = 50.40$ bar, Z_C=0.281.

$$T_r = \frac{298}{282.3} = 1.056 \qquad P_r = \frac{12}{50.4} = 0.2381$$

From Equation Sheet

$$B^\circ = 0.0833 - \frac{0.422}{T_r^{1.6}} = 0.0833 - \frac{0.422}{(1.056)^{1.6}} = -0.3035$$

$$B' = 0.139 - \frac{0.172}{T_r^{4.2}} = 0.0022$$

$$Z^\circ = 1 + B^\circ \frac{P_r}{T_r} = 1 + (-0.3035)\frac{0.2381}{1.056} = 0.9316$$

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$$z' = 0.0022\left(\frac{0.2381}{1.056}\right) = 4.96 \times 10^{-4}$$

$$Z = Z^{\circ} + \omega Z' = 0.9316 + (0.087)(4.96 \times 10^{-4})$$

$$\boxed{Z = 0.9316}$$

$$V = \frac{ZRT}{P} = \frac{(0.9316)(8.314)(298)}{(1200)}$$

$$\boxed{V = 1.92 \ \text{L mol}^{-1}}$$

8 minutes

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4. Estimate the residual properties, H^R and S^R using the method of your choice for methane at 250 K and 90 bar.
For methane (From Table B of text) (10)
Molar mass = 16.043, acentric factor, $\omega = 0.012$, $T_C = 190.6$ K, $P_C = 45.99$ bar, $Z_C = 0.286$ and $V_C = 98.6\ \text{cm}^3\ \text{mol}^{-1}$

$$\frac{H^R}{RT_c} = P_r\left[B^0 - \frac{0.675}{T_r^{1.6}} + \omega\left(B^1 - \frac{0.722}{T_r^{4.2}}\right)\right]$$

$$\frac{S^R}{R} = -P_r\left[\frac{0.675}{T_r^{2.6}} + \omega\left(\frac{0.722}{T_r^{5.2}}\right)\right]$$

$$T_r = \frac{250}{191} = 1.309 \qquad P_r = 1.957$$

$$B^0 = 0.083 - \frac{0.422}{T_r^{1.6}} = 0.083 - \frac{0.422}{(1.309)^{1.6}} = -0.1913$$

$$B^1 = 0.139 - \frac{0.172}{T_r^{4.2}} = 0.139 - \frac{0.172}{(1.309)^{4.2}} = -0.1110$$

$$H^R = (8.314)(190.6)(1.957)\left[-0.1913 - \frac{0.675}{(1.309)^{1.6}} + 0.012\left(-0.1110 - \frac{0.722}{(1.309)^{4.2}}\right)\right]$$

$$= (8.314)(190.6)(1.957)\left[-0.1913 - 0.4387 + 0.012(-0.1110 - 0.2330)\right]$$

$$= -1966.5\ \text{J/mol.}$$

$$S^R = -(8.314)(1.957)\left[\frac{0.675}{(1.309)^{2.6}} + 0.012\left(\frac{0.722}{(1.309)^{4.2}}\right)\right]$$

$$= -5.5 \ \frac{J}{mol\,K}$$

15 min

Could also use Redlich - Kwong

3. Using the Gibbs Equations the Maxwell's Equations and the fundamental definitions of C_v and dU to show the derivation of

$$dS = C_V \frac{dT}{T} + \left(\frac{\partial P}{\partial T}\right)_V dV$$

(Hint start with the expansion: $dS = \left(\frac{\partial S}{\partial T}\right)_V dT + \left(\frac{\partial S}{\partial V}\right)_T dV$) (10)

$$dS = \left(\frac{\partial S}{\partial T}\right)_V dT + \left(\frac{\partial S}{\partial V}\right)_T dV$$

from Maxwell's Equations subst. into second term

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$$

giving

$$dS = \left(\frac{\partial S}{\partial T}\right)_V dT + \left(\frac{\partial P}{\partial T}\right)_V dV \quad (\text{II})$$

assume const V (dV=0) and use Gibbs equation (for Term 1)

$$dU = TdS + \cancel{PdV}^{\,0}$$

$$dU = TdS$$

substitute $dU = C_v dT$

$$\Rightarrow \quad C_v dT = TdS$$

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divide by dT

$$C_v = T\left(\frac{\partial S}{\partial T}\right)_V$$

re-arrange

$$\frac{C_v}{T} = \left(\frac{\partial S}{\partial T}\right)_V$$

substitute into original equation (II)

to give

$$\boxed{dS = C_v\frac{dT}{T} + \left(\frac{\partial P}{\partial T}\right)_V dV}$$

(7 minutes)

4. Consider the heating of a house by a furnace, which serves as a heat-source reservoir at a high temperature T_F. The house acts as a heat-sink reservoir at constant temperature T, and heat Q must be added to the house during a particular time interval to maintain this temperature. All the heat Q could be transferred directly from the furnace to the house, as per the usual practice. However, a third heat reservoir is also available, namely, the surroundings at temperature, T_s, which can serve as another heat source (using a heat pump). Given that the temperature of the furnace is $T_F = 840$ K, the temperature of the house is $T = 295$ K and the temperature of the surroundings is $T_s = 270$ K and Q=1000 kJ, determine the minimum amount of heat, Q_F from the furnace which must be extracted from the heat-source reservoir (furnace) at T_F to maintain the temperature T. No other sources of energy are available and the house can be considered a closed system with no air exchange. Steady-state energy and entropy balance should both be used.

(Hint: Recall that for isothermal transfer of heat, $\Delta S = \frac{Q}{T}$ and that for a reversible process $S_{gen} = 0 = \Delta S_{Total}$) (10)

Given $Q = -1000\,J$ (lost by house)

$T_F = 840\,K$

$T_s = 270\,K$

$T = 295\,K$

Energy balance (s.s.)

$$Q_F + Q_s + Q = 0$$

Solve for Q_s:

$$Q_s = -Q - Q_F$$

$$\boxed{Q_s = 1000 - Q_F}$$

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Entropy Balance (s.s.)

$$\cancel{\Delta(\dot{m}S)_{fs}}^{\,0} = \sum_j \frac{\dot{Q}_j}{T_{o,j}} + \cancel{\dot{S}_{gen}}^{\,0}$$

no flowing streams.

minimum amount of heat for reversible process

$$\sum_j \frac{\dot{Q}_j}{T_{o,j}} = 0$$

for a finite amount of time

$$\frac{Q_F}{T_F} + \frac{Q_S}{T_S} + \frac{Q}{T} = 0$$

substitute from energy balance and $Q = 1000$

$$\frac{Q_F}{T_F} + \frac{(1000 - Q_F)}{T_S} + \frac{-1000}{T} = 0$$

$$Q_F\left(\frac{1}{T_F} - \frac{1}{T_S}\right) + 1000\left(\frac{1}{T_S} - \frac{1}{T}\right) = 0$$

$$Q_F = -1000 \frac{\left(\frac{1}{T_S} - \frac{1}{T}\right)}{\left(\frac{1}{T_F} - \frac{1}{T_S}\right)} = -1000 \frac{\left(\frac{1}{270} - \frac{1}{295}\right)}{\left(\frac{1}{840} - \frac{1}{270}\right)}$$

$$= 125 \text{ kJ}$$

5 minutes