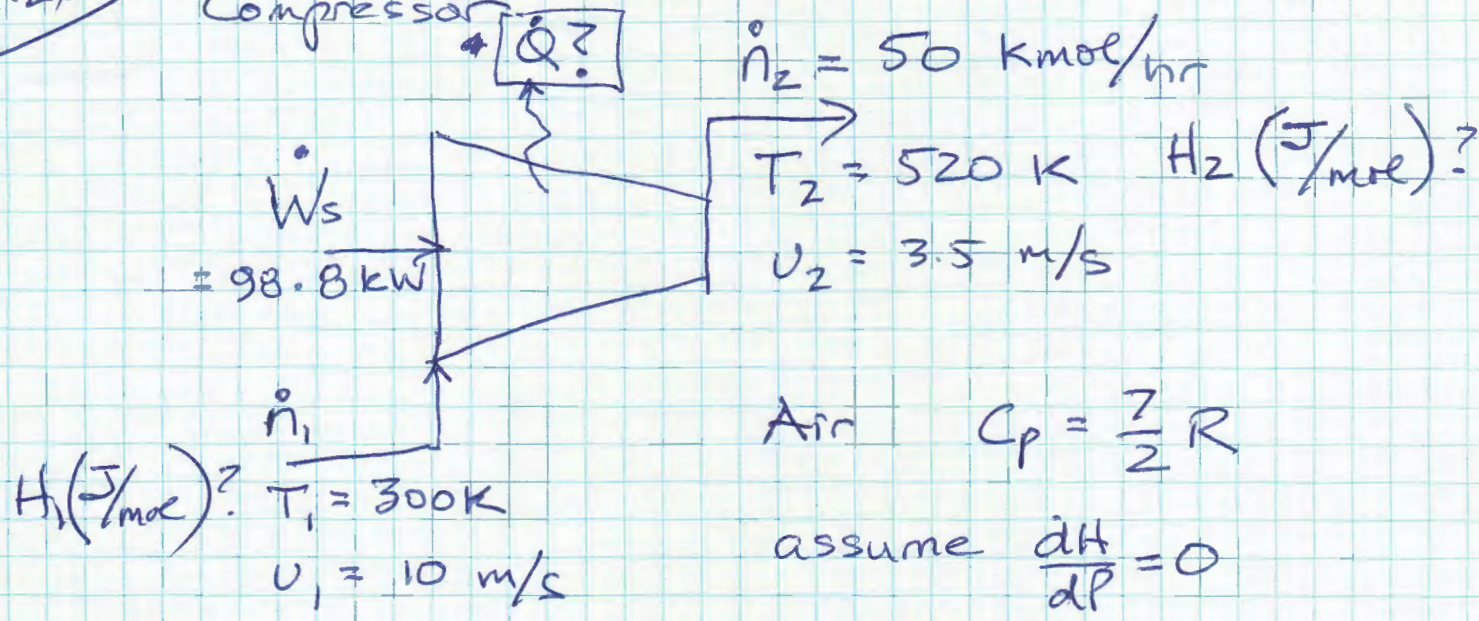


2.26

PS1

Compressor



Find $\dot{Q}?$

e.g., enthalpy does not vary with pressure

Equations:

Definition of C_p . (text p41, Eq'n 2.22 for an ideal gas)

$$C_p \equiv \left(\frac{dH}{dT} \right)_{IG}$$

$$dH = C_p dT$$

$$\Delta H = C_p \Delta T$$

(even if not constant pressure)

Mass Balance (text p46, Eq'ns 2.25, 2.26)

$$\frac{dm_{cv}}{dt} + \Delta(\dot{m})_{fs} = 0$$

but steady-state so

$$\Delta(\dot{m})_{fs} = \Delta(\rho u A)_{fs} = 0$$

$$\dot{m} = \frac{u_1 A_1}{v_1} = \frac{u_2 A_2}{v_2} \quad \left(\text{for } | \text{ entering} \right. \\ \left. + | \text{ exiting stream} \right)$$

Energy Balance (Pg 48)

$$\frac{d(mU)_{cv}}{dt} + \Delta \left[\left(H + \frac{1}{2}u^2 + zg \right) \dot{m} \right]_{fs} = \dot{Q} + \dot{W}$$

(Eq'n 2.28)

but we have steady-state hence
 Eq'n 2.27

$$\Delta \left[\left(H + \frac{1}{2}u^2 + zg \right) \dot{m} \right]_{fs} = \dot{Q} + \dot{W}_s$$

↳ shaft work

(Eq'n 2.30)

Assume $\Delta zg = 0$ (the compressor is stationary)

then

$$\Delta \left[\left(H + \frac{1}{2}u^2 \right) \dot{m} \right]_{fs} = \dot{Q} + \dot{W}_s$$

need to deal with mass flow and mole flow consistency. Kinetic energy requires "mass" but C_p is in J/mole K and flow is given as 50 kmol/hr.

Assume molar mass of air = 29 g/mole

or
 29 kg/kmol.

Now

$$\left[(H_2 - H_1) + \left(\frac{u_2^2}{2} - \frac{u_1^2}{2} \right) M_{AIR} \right] \dot{n} - \dot{W}_S = \dot{Q}$$

$$\Delta H = (H_2 - H_1) = C_p (T_2 - T_1) \text{ where } C_p = \frac{7}{2} R$$

$$\begin{aligned} \Delta H &= \frac{7}{2} 8.314 \frac{\text{kJ}}{\text{kmol K}} (520 \text{ K} - 300 \text{ K}) = \frac{7}{2} (8.314 \frac{\text{J}}{\text{mol K}}) \\ &= 6.402 \times 10^3 \frac{\text{kJ}}{\text{kmol}} = \frac{7}{2} (8.314 \frac{\text{kJ}}{\text{kmol K}}) \end{aligned}$$

$$\begin{aligned} \left(\frac{u_2^2}{2} - \frac{u_1^2}{2} \right) M_{AIR} &= \left(\frac{3.5^2 \text{ m}^2}{2 \text{ s}^2} - \frac{10^2 \text{ m}^2}{2 \text{ s}^2} \right) \cdot 29 \frac{\text{kg}}{\text{kmol}} \\ &= -43.9 \cdot 29 \left(\frac{\text{kg} \cdot \text{m}^2}{\text{s}^2} \right) / \text{kmol} \end{aligned}$$

$$1 \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2} = 1 \text{ J}$$

$$= -1272 \text{ J} / \text{kmol}$$

$$= -1.272 \text{ kJ} / \text{kmol}$$

subst.

$$\left[6.402 \times 10^3 - 1.272 \right] 50 \frac{\text{kmol}}{\text{h}} - \dot{W}_S = \dot{Q}$$

$$320036 \text{ kJ/h} - \dot{W}_S = \dot{Q}$$

But 1 kW = 1 kJ/s need to correct units.

PS 4

$$320036 \frac{\text{kJ}}{\text{h}} \times \frac{1 \text{ h}}{3600 \text{ s}} = 88.9 \frac{\text{kJ}}{\text{s}}$$
$$= 88.9 \text{ kW}$$

Now

$$88.9 \text{ kW} - \dot{W}_s = \dot{Q}$$

$$\dot{W}_s = 98.8 \text{ kW} \quad (\text{given})$$

$$\therefore \dot{Q} = 88.9 - 98.8 = -9.9 \text{ kW}$$

The compressor is giving off 9.9 kW of heat in compressing 50 kmol/hr.

NOTE: Kinetic energy had very little effect. \Rightarrow Use for problem 2.42

Steady State Energy Balance

$$\Delta \left[\left(H + \frac{1}{2} u^2 + z g \right) \dot{m} \right]_{fs} = \dot{Q} + \dot{W}_s \Rightarrow \Delta [H \dot{m}]_{fs} = \dot{W}_s$$

$$2.42 \quad \dot{m} := 4.5 \frac{\text{kg}}{\text{s}}$$

$$H_1 := 761.1 \frac{\text{kJ}}{\text{kg}}$$

$$H_2 := 536.9 \frac{\text{kJ}}{\text{kg}}$$

Assume that the compressor is adiabatic ($\dot{Q} = 0$). Neglect changes in KE and PE.

$$\dot{W}_s = \dot{m} (H_2 - H_1) \text{ all given}$$

$$\dot{W} := \dot{m} \cdot (H_2 - H_1)$$

$$\dot{W}_s = -1.009 \times 10^3 \text{ kW}$$

$$\dot{W} = -1.009 \times 10^3 \text{ kW}$$

$$\text{Cost} := 15200 \cdot \left(\frac{|\dot{W}|}{\text{kW}} \right)^{0.573}$$

$$\text{Cost} = 799924 \text{ dollars Ans.}$$

↑ Single cost function
that predicts cost in dollars
for a given power in kW.