Instructions:

1. This is a 50 minute examination.

2. Please write your student number instead of your name on all answer booklets and sheets submitted.

3. Please state all assumptions made.

4. Calculators are permitted.

5. This is an open-book examination. Textbooks, course notes, assignment and midterm solutions are allowed.

6. Please answer all questions.

7. There are three (3) pages in the question booklet.

8. Good luck!

Please note: “If the instructor is unavailable in the examination room and if doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper a clear statement of any assumptions made.”
**Question 1**

Find the poles and zeros of the following rational polynomial functions. Identify the dominant (or slow) poles and dominant (or slow) stable and unstable zeros. For all stable processes, sketch the unit step response and compute the steady-state gain.

1. 
   \[ G(s) = \frac{(s - 0.7)}{1.1(s + 0.92)(s + 12.2)(s^2 + 1.5s + 1)} \]

   **Solution:**
   - Poles: -0.92, -12.2, -0.75 ± 0.6614j (2 marks, 0.5 each)
   - Zeros: 0.7 (0.5 marks)
   - Dominant Stable pole: -0.75 ± 0.6614j (0.5 mark)
   - Dominant Unstable Zero: 0.77 (0.5 mark)
   - Sketch shows underdamped (0.5) and inverse response (0.5) with negative steady-state gain (0.5).

2. 
   \[ G(s) = \frac{-2(s - 1)}{s^2 + 4s + 3} \]

   **Solution:**
   - Poles: -1, -3 (2 marks, 1 each)
   - Zeros: 1 (0.5 marks)
   - Dominant Stable pole: -1 (0.5 mark)
   - Dominant Unstable Zero: 1 (0.5 mark)
   - Sketch shows underdamped (0.5) and inverse response (0.5) with positive steady-state gain (0.5).

**Question 2**

Consider the following second order system:

1. 
   \[ G(s) = \frac{2.3(-2s + 3.3)}{1.2s^2 + 0.1s + 1} \]
2.

\[ G(s) = \frac{5}{4s^2 + 5s + 2} \]

For each system:

- Compute the steady-state gain, \( K \), the natural period of oscillation, \( \tau \) and the damping coefficient \( \xi \).
- Identify the system as overdamped, critically damped or underdamped.

**Solution:** System 1:

- \( K = 7.59 \), \( \tau = 1.0954 \), \( \xi = 0.0456 \) (3 marks)
- The system is underdamped (2 marks).

System 2:

- \( K = 2.5 \), \( \tau = \sqrt{2} \), \( \xi = 5\sqrt{2}/4 = 1.7678 \) (3 marks)
- The system is overdamped (2 marks).
Question 3

The following model is given:

\[ G(s) = \frac{s - 3}{(s + 4)(s - 2)} \]

1. Using direct synthesis, design a controller that stabilizes this process. Pick your desired complementary sensitivity \((T(s))d\) to be of the form:

\[ (T(s))d = \frac{c(s)}{(s + 1)^r} \]

where \(c(s)\) and the integer \(r\) must be chosen such that the resulting controller is biproper and the closed-loop system is stable with zero steady-state error. Assess the stability of the closed-loop system.

**Solution:**

Design \((T(s))d\) such that \((T(0))d = 1\), \((T(3))d = 0\) and \((T(2))d = 1\). (3 marks)

Required \((T(s))d\) has the form:

\[ (T(s))d = \frac{(\eta s + 1)(-\frac{1}{3}s + 1)}{(s + 1)^3} \]

(1 marks for \(c(s)\) and 1 mark for \(r = 3\).

Solve for \(\eta\) gives:

\[ \frac{(2\eta + 1)(-\frac{2}{3} + 1)}{(2 + 1)^3} = 1 \]

or

\[ \eta = \frac{(27(3) - 1)}{2} = 40 \]

(1 mark for \(\eta\))

\[ (T(s))d = \frac{(40s + 1)(-s + 3)}{3(s + 1)^3} \]

Solve for \(C(s)\)

\[ C(s) = \frac{(s + 4)(s - 2) - \frac{(40s + 1)(-s + 3)}{3(s + 1)^3}}{s - 3 - \frac{(40s + 1)(-s + 3)}{3(s + 1)^3}} = -(s + 4)(s - 2) \cdot \frac{3(40s + 1)}{3(s + 1)^3} \cdot \frac{3(s + 1)^3}{3(s + 1)^3} \]

\[ = \frac{(s + 4)(s - 2)(40s + 1)}{3(3(s + 1)^3 - (40s + 1)(-s + 3))} = \frac{(s + 4)(s - 2)(40s + 1)}{3(3s^3 + 49s^2 - 110s)} = \frac{(s + 4)(40s + 1)}{3s(s + \frac{55}{3})} \]

(6 marks for the controller)
For this controller, the $A_c(s)$ is given by:

$$(s + 4)(s - 2)s(s + \frac{55}{3}) + (s - 3)(s + 4)(40s + 1) = (s + 4)(s^3 + 3s^2 + 3s + 1) = (s + 4)(s + 1)^3$$

All poles are on the OLHP, the closed-loop system is stable. (2 marks)

2. Using pole placement, design a controller with integral action that stabilizes the system. Pick your design closed-loop characteristic polynomial to be of the form:

$$A_c(s) = (s + 4)(s + 1)^r$$

where the integer $r$ must be chosen such that the resulting controller is bi-proper. **Solution:**

Since $\deg(A(s)) = 2$, forcing integral action requires $\deg(A_c) = 4$. Therefore $r = 3$. (2 marks)

Two alternatives here:

(2.1) The required $A_c$ is achieved by the controller derived above. That is, $C(s) = -\frac{(s+4)(40s+1)}{s(s+\frac{55}{3})}$. (12 Marks)

(2.2) Solve the problem again:

By pole placement, $\deg(P(s)) = 2$ and $\deg(s\bar{L}(s)) = 2$. Therefore, $P(s) = (s+4)(p_1s+p_0)$ (2 marks) and $\bar{L}(s) = \ell_1s + \ell_0$. (2 marks)

The pole placement equation is given by:

$$(s + 4)(s^3 + 3s^2 + 3s + 1) = (s + 4)(s - 2)s(\ell_1s + \ell_0) + (s - 3)(s + 4)(p_1s + p_0)$$

$$(s^3 + 3s^2 + 3s + 1) = (s - 2)(\ell_1s^2 + \ell_0s) + p_1s^2 + (-3p_1 + p_0)s - 3p_0$$

$$(s^3 + 3s^2 + 3s + 1) = (\ell_1s^3 + (-2\ell_1 + \ell_0 + p_1)s^2 + (-2\ell_0 - 3p_1 + p_0)s - 3p_0)$$

Solving gives: $\ell_1 = 1$, $\ell_0 = \frac{55}{3}$, $p_0 = -\frac{1}{3}$, $p_1 = -\frac{40}{3}$. (8 marks, 2 each)

3. Compare the controllers obtained from both approaches. **Solution:**

They are the same (2 marks).