The goal of this tutorial is to show you how to use simulink as a means to visualize the solution to a differential equation.

1. Consider the Laplace transformed variable of question (1.2) from Tutorial 3:

\[ F_1(s) = \frac{10}{s(s + 1)(s + 10)} \]

A “transfer function” is an input/output relationship which you will become very familiar with in this course. Transfer functions are written as

\[ \frac{Y(s)}{U(s)} = \frac{N(s)}{D(s)} \]

Where \( Y(s) \) and \( U(s) \) are the Laplace transforms of the output and input variables, respectively and \( N(s) \) and \( D(s) \) are the numerator and denominator polynomials in the Laplace variable, \( s \).

\( F_1(s) \) can be thought of as \( Y(s) \) when a unit-step input \((U(s) = \frac{1}{s})\) is applied to the transfer function

\[ \frac{Y(s)}{U(s)} = \frac{10}{(s + 1)(s + 10)} \]

Simulink has all of the necessary tools to simulate the responses of systems described by transfer functions in a very convenient manner.

To launch simulink, type simulink in the MATLAB command window. In the Simulink Library Browser, click File > New > Model to begin working on a new model. To import a transfer function block, look under the Continuous menu and find Transfer Fcn. Click and drag this block into the empty model workspace.

Double-click on the Transfer Fcn block. This will open a dialogue box where you can specify the numerator and denominator polynomials.

Simulink interprets the arguments in the “Numerator coefficients” and “Denominator coefficients” fields as vectors whose entries correspond to coefficients of \( N(s) \) and \( D(s) \) in descending powers of \( s \). For this example, the numerator of our transfer function is simply 10, so enter “10” into the square parentheses in the “Numerator coefficients” field. For the denominator field, enter [1 11 10], since this corresponds to \( s^2 + 11s + 10 \), which is the denominator polynomial of \( Y(s)/U(s) \).
You will notice that the transfer function block has a port to accept an input. For this example we wish to supply a unit step input. In the Simulink Library Browser, go to Sources and click and drag the Step block into your model workspace. Connect the two blocks by clicking on the output of the Step block and dragging it to the input of the Transfer Fcn block (or, you can left-click on the Step block, hold Ctrl, and left-click on the Transfer Fcn block).

To visualize the output, go back to the Simulink Library Browser, and under the Sinks menu, click and drag the Scope block to the model workspace. Connect this block to the output of the Transfer Fcn block. Set the simulation length to 10 seconds (if it isn’t already set to that) and hit the play button to execute the simulation. Double click on the Scope block to see the results. If the outputs are badly scaled, click on the binoculars icon to automatically scale the y-axis.

**Questions:**

- Does this trajectory agree with the causal $f(t)$ of $F(s)$ that you found in Tutorial 3?

- Replace the denominator polynomial with $s^2 + 2s + 10$ and re-run the simulation. Are the roots real or complex? What has happened to the output as a result?

- Now consider the denominator polynomial $D(s) = (s - \frac{1}{10})$. Re-run the simulation, what do you notice about the value of the output as time progresses? Will the output ever reach a steady-state value? Why or why not? (use the solution to the differential equation as a means to explain this)

- Consider a function

  $$F(s) = \frac{1}{s^2 - 0.5s + 4}$$

  simulate the unit step response of $F(s)$ in Simulink. Analyze the response. How different is it from above responses? Why?

2. Consider the Laplace transformed variable:

  $$F(s) = \frac{12}{(s + 8)(s + 12)}$$

  - Expand the above expression using partial fraction expansions.

  - Using simulink, simulate the unit step response for $F(s)$ and its expanded form. To simulate the expanded form, you will need to split $F(s)$ into the sum of two individual transfer functions (both will have first order denominators). Use the Add block under the Math Operations menu to accomplish this.

  - Compare the output of $F(s)$ and output of its expanded form. What is the difference between the two responses?