**Problem Statement**

In this tutorial, we revisit the dynamical model of the gravity tank that we studied in Tutorial 1. A schematic diagram of the process is shown below.

The dynamics of the process are given by:

\[
\frac{dh}{dt} = \frac{F_o}{A} - \frac{A_p v}{A} \\
\frac{dv}{dt} = \frac{hg}{L} - \frac{K_f v^2}{\rho A_p L}
\]

where \(A\) and \(A_p\) are the cross-sectional areas of the tank and the outlet pipe, \(g\) (=9.8 m/sec\(^2\)) is the gravitation constant, \(L\) is the length of the outlet pipe, \(h\) is the height of liquid in the tank, \(v\) is the average velocity of the fluid in the outlet pipe, \(K_f\) is the pipe friction constant, \(\rho\) is the density of the fluid and \(F_o\) is the inlet flowrate of fluid to the tank. It is assumed that the tank and the outlet pipe are cylindrical.

The tank has a radius \(R\) of 2 m. The inner radius of the pipe, \(r\), is 0.125 m. The outlet pipe is 10 m in length. The density of the fluid is 1000 kg/m\(^3\). The friction constant is 4.0.

We already know how to simulate the dynamic behavior of the nonlinear process using the SIMULINK differential equation editor (DEE).

- Prepare the nonlinear simulation of the gravity tank following the steps outlined in tutorial 1.

- Perform the following test.

  1. Assuming an inlet flow of 0.45 m\(^3\)/sec, find the steady-state values for the height of liquid in the tank and the outlet pipe fluid velocity. Report the steady-state value and show the resulting plots of \(h\) and \(v\).
2. Starting for this steady-state, perform a 0.25 m$^3$/sec step increase and a 0.25 m$^3$/sec step decrease in inlet flowrate. Report the two new sets of steady-state values. Generate a first plot showing both step responses for the level and a second showing the step responses for the velocity. Briefly comment on the extent of nonlinearity of this process.

We will now linearize the gravity tank about the steady-state conditions corresponding to an inlet flow of 0.45 m$^3$/sec.

- Using 1st Taylor series expansion, write the linear approximation of the nonlinear equations and express the dynamics in deviation form.
• Compute the transfer function relating the tank level and the inlet flow.

• As for the nonlinear model, perform a 0.25 m$^3$/sec step increase and a 0.25 m$^3$/sec step decrease in inlet flowrate. Report the two new sets of steady-state values. Generate a first plot showing both step responses for the level and a second showing the step responses for the velocity.

• Compare the results with the nonlinear simulation and briefly comment on the extent of nonlinearity of this process.