• **Question 1**

The filter type is *band-pass*, as seen by the constant gain region between $\omega_{c,1} \approx 1.2 \text{rad/s}$ and $\omega_{c,2} \approx 1,100 \text{rad/s}$.

Hence, the *bandwidth* is $\omega_{c,2} - \omega_{c,1} \approx 1,100 \text{rad/s}$.

• **Question 2**

1. $G_0(s)C(s)$ contains no unstable poles, but does contain a pole at $s = 0$. Therefore, the modified Nyquist path is used. Since $G_0(s)C(s)$ is open-loop stable, we require zero encirclements of the critical point $(-1,0)$. Inspecting the Nyquist plot, we see that the critical point $(-1,0)$ is encircled once by the plot as it extends around the left-hand side of the plot. Therefore, the closed-loop system will be *unstable* with the proposed controller.

2. $G_0(s)C(s)$ contains one unstable pole (at $s = +1$). For closed-loop stability, we require that the Nyquist plot encircle the critical point $(-1,0)$ *once* in the counter-clockwise direction. This is the case, and so the closed-loop system will be stable.

3. $G_0(s)C(s)$ is open-loop stable, so for closed-loop stability we require no encirclements of the critical point $(-1,0)$. Examining the Nyquist plot, we see that there are several encirclements. We therefore conclude that the closed loop system will be unstable.

The remaining answers are summarized:

4. $N = -2, P = 2$
   
   So $Z = N + P = 0$, the closed-loop system is *stable*.

5. $N = 0, P = 2$
   
   So $Z = N + P = 2$, the closed-loop system is *unstable*.

6. $N = 1, P = 1$
   
   So $Z = N + P = 2$, the closed-loop system is *unstable*.

7. $N = 1, P = 1$
   
   So $Z = N + P = 1$, the closed-loop system is *unstable*.

• **Question 3**

   As the phase margin is approximately 35 degrees. The system is stable.

• **Question 4** We are given a plant described by

$$G_0(s) = \frac{(s + 4)}{(s + 1)(s + 3)}.$$
and are asked to design a commercial PID (i.e. PID w/ 1st order filter) which will guarantee disturbance rejection for disturbance signals with energy in the band \([0,5]\) rad/s.

Since \(G_0(s)\) does not contain delay or unstable poles/zeros, we will use the straightforward method of direct synthesis. For arbitrary controller \(C(s)\), the closed-loop transfer function (“complementary sensitivity function”) is given by:

\[
T(s) = \frac{G_0(s)C(s)}{1 + G_0(s)C(s)}
\]

We would like \(T(s) \approx 1\) for disturbance signals with frequency between \(\omega = [0, 5]\) rad/s. Since we are designing \(C(s)\) via direct synthesis, define our desired closed-loop system as

\[
T(s) = \frac{1}{\tau_{cl}s + 1}.
\]

We must specify \(\tau_{cl}\) so that \(T(s) \approx 1\) for frequencies \(\omega = [0, 5]\) rad/s. We know that for first order systems, the breakpoint frequency as observed on a Bode plot of our system will occur at

\[
\omega_{bp} = \frac{1}{\tau_{cl}}.
\]

Therefore, we choose \(\tau_{cl}\) such that the amplitude ratio of \(T(s)\) for frequencies \(\omega = [0, 5]\) rad/s remains at 1. An appropriate choice then is \(\tau_{cl} = 1/10\) (a slight over-design). Now we proceed with the direct synthesis method:

\[
C(s) = \frac{\frac{1}{\tau_{cl}s+1}}{(s+4)\left(1 - \frac{1}{\tau_{cl}s+1}\right)},
\]

\[
= \frac{(s+1)(s+3)}{(\tau_{cl}s+1)(s+4)\left(1 - \frac{1}{\tau_{cl}s+1}\right)},
\]

\[
= \frac{(s+1)(s+3)}{(s+4)(\tau_{cl}s+1 - 1)},
\]

\[
= \frac{(s+1)(s+3)}{(s+4)\tau_{cl}s},
\]

\[
= \frac{1}{(s+4)} \left( \frac{s^2 + 4s + 3}{\tau_{cl}s} \right),
\]

\[
= \frac{1}{(s+4)} \left( \frac{4}{\tau_{cl}} \right) \left[ 1 + 3/4 \frac{1}{s} + 1/4s \right].
\]
Figure 1: Bode plot for $T(s)$ with $\tau_{cl} = 0.1$. 