Question 1

Consider the linear system whose open-loop dynamics are described by the transfer function:

\[ G_0(s) = \frac{1}{(s + 3)} \]

The system is subject to an unmeasured disturbance \( d_g(t) = a + \sin(2t + \phi) \) where \( a \) and \( \phi \) are unknown constants.

1. Using pole placement, design a controller that stabilizes the closed-loop control system and achieves zero steady-state output error.

   Consider a closed-loop characteristic polynomial of the form:

   \[ A_{cl}(s) = (s + 3)(s + 1)(s + 2)^r \]

   where \( r \) is an integer to be assigned in the design of the controller.

2. Compute the sensitivity function.

Question 2

Consider the linear system whose open-loop dynamics are described by the transfer function:

\[ G_0(s) = \frac{1}{(s + 4)} \]  \hspace{1cm} (1)

The system is subject to an unmeasured disturbance \( d_g(t) = a + \sin(t + \phi) \) where \( a \) and \( \phi \) are unknown constants.
1. Using pole placement, design a controller that stabilizes the closed-loop control system and achieves zero steady-state output error.

Consider a closed-loop characteristic polynomial of the form:

\[ A_{cl}(s) = (s + 4)(s + 1)^r \]

where \( r \) is an integer to be assigned in the design of the controller.

2. Compute the sensitivity function.

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**Question 3**

Consider the transfer function,

\[ G_0(s) = \frac{5}{s + 1} \]  \hspace{1cm} (2)

1. Assuming that a general disturbance \( D_g(s) \) is not available for measurement, design a control system that guarantees zero steady-state error when the control system is subject to a disturbance \( D_g(s) \) with disturbance-generating polynomial:

\[ \Gamma_d(s) = s(s^2 + 16). \]  \hspace{1cm} (3)

Use pole-placement with a characteristic polynomial given by: \((s + 1)^r\) where \( r \) is an integer to be chosen.

2. What principle is invoked in this case? What happens if the disturbance enters the system at frequencies other than \( \omega = 0 \) and \( \omega = 4 \) [rad/s]?

3. Compute the complementary sensitivity for this control system. Will the process reject disturbances in the band \( B_d = [0, 1] \text{ rad/s} \).

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**Question 4**

Consider the linear system whose open-loop dynamics are described by the transfer function:

\[ G_0(s) = \frac{10}{s(s + 2)} \]  \hspace{1cm} (4)

The system is subject to an unmeasured disturbance \( d_g(t) = a + \sin(t + \phi) \) where \( a \) and \( \phi \) are unknown constants.
1. Using pole placement, design a controller that stabilizes the closed-loop control system and achieves zero steady-state output error.

Consider a closed-loop characteristic polynomial of the form:

\[ A_{cl}(s) = (s + 2)(s + 1)^r \]

where \( r \) is an integer to be assigned in the design of the controller.

2. Compute the sensitivity function.

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**Question 4**

Consider the process model:

\[ G_o(s) = \frac{-s + 7}{(s + 2)(s + 3)} \]

(a) Using pole placement, design a biproper controller that yields the closed-loop characteristic polynomial given by:

\[ A_{cl}(s) = (s + 2)(s + 3)(2s + 1)^r \]

where \( r \) is an integer to be assigned. Moreover, we seek to reject a disturbance of the form:

\[ d(t) = K_0 + K_1 \sin(3t + \phi) \]

where \( K_0, K_1 \) and \( \phi \) are unknown nonzero constants.

(b) Assume that the actual plant model is given by:

\[ G(s) = \frac{s + 1}{(s + 2)(s + 3)} \]

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**Question 5**

Consider the dynamical system depicted in Figure 1.

(a) Assuming that the disturbance \( D_g(s) \) is available for measurement, design a control system that reduces the effect of \( D_g(s) \) and achieves zero steady-state error for any reference output signal \( R(s) \) when the control system is subject to any change in \( D_g(s) \). Identify the controller configuration clearly using a block diagram of the control system. (Note that a feedback controller is necessary to ensure that \( Y(s) \) tracks \( R(s) \). Use pole placement to design a biproper feedback controller with integral action that stabilizes the system.)
Figure 1: Linear System for Question 2.

(b) Assuming that the disturbance $D_g(s)$ is not available for measurement, design a control system that guarantees zero steady-state error when the control system is subject to a disturbance $D_g(s)$ with disturbance-generating polynomial:

$$\Gamma_d(s) = s(s^2 + 10).$$

(5)

What principle is invoked in this case? What happens if the disturbance enters the system at frequencies other than $\omega = 0$ and $\omega = \sqrt{10}$ [rad/s]?